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Additional Sweep-Frequency Impedance Measuring Techniques

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The use of a Delay Line with a Sweep-Frequency Generator for impedance measurement and adjustment, as described in Technical Newsletter #2, provides a powerful measuring tool, most useful for wide bandwidths. When the impedance to be measured is part of a resonant system with a bandwidth less than about 10% of the center frequency, the delay line technique is less useful. The long line necessary to get the needed resolution has so much loss that measurement accuracy is reduced. The techniques described below are particularly useful with narrow-band circuits.

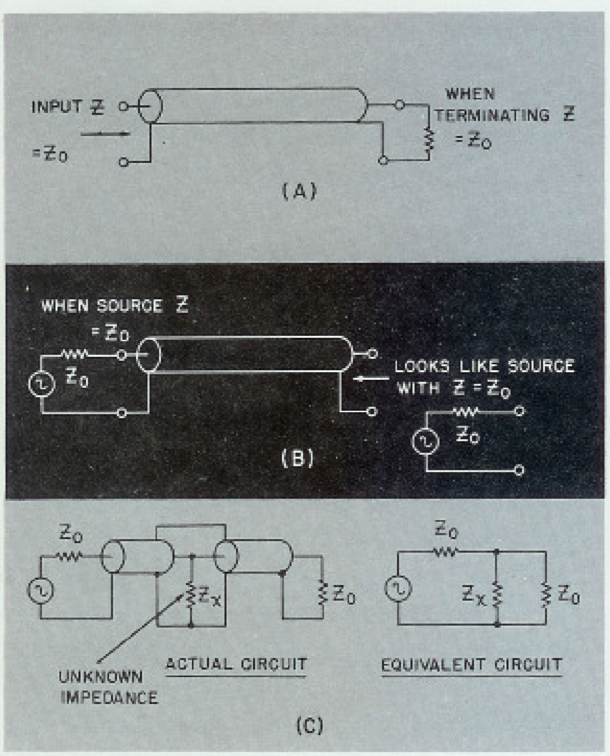


Figure 1—Measurement by Bridging Loss

MEASUREMENT BY BRIDGING LOSS

It is basic in transmission line theory that the impedance seen looking into a section of line which is terminated at the far end in its characteristic impedance is the characteristic impedance—a pure resistance for low-loss lines. (See Figure 1A.) Similarly, the equivalent circuit seen looking back into a line fed from a matched source is a constant voltage in series with a resistor equal to Z_0 (Fig. 1B).

It follows that the circuit seen at any point along a matched system where the line from the source joins the line to the load is equivalent to a resistive source feeding a resistive load.

When a given impedance is "bridged," i.e. connected across an otherwise matched system, the loss of energy transferred to the load is the same as the loss that is calculated when bridging that same impedance across the load in the equivalent circuit (Figure 1C).

In the general case where the unknown impedance had both resistance and reactance, it would be necessary to know both the magnitude and phase angle of the loss introduced to determine the unknown constants. Where suitable equipment is available to measure both quantities, a chart has been worked out relating them to the constants of the bridging load (see "A Precise Sweep-Frequency Method of Vector Impedance Measurement" by D. A. Alsberg, Proc. I.R.E. Nov. 1951. PP 1393-1400).

In many practical situations it is known, from other considerations, that the unknown is either a pure resistance or a pure reactance. In these cases the magnitude of the unknown is determined by a simple attenuation measurement. The relationships can be calculated as shown in Appendix I; the results can be drawn up as nomographs relating bridging loss to attenuation. Two such charts have been made, and are illustrated on page 6 as Figure 9 (for systems with 50 ohm impedance) and Figure 10 (for 75 ohm systems).

MEASUREMENT BY SERIES LOSS

"Bridging" is not the only way in which an unknown impedance can be connected into a matched system. It is also possible to open the center conductor at a junction and insert the unknown in series, as indicated in Figure 4.

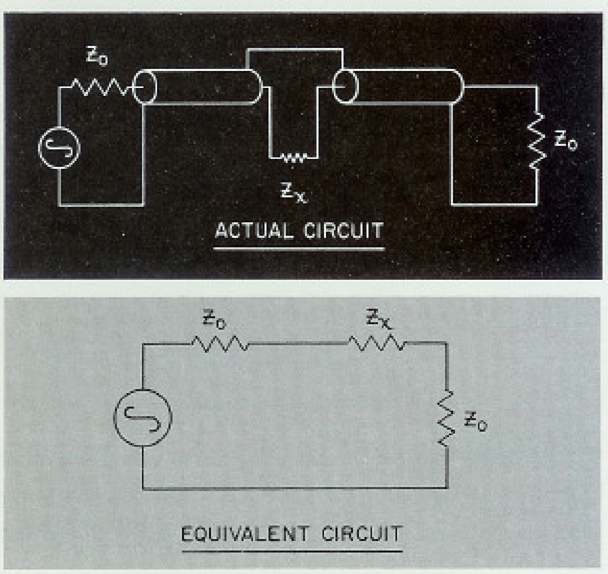


Figure 2—Measurement by Series Loss

The loss that is introduced is a measure of the impedance in this case also. For unknowns that are purely resistive or purely reactive, the loss can be calculated (see Appendix I, page 8) and the resulting relationship drawn up in the form of Nomographs: (See page 7, Figure 11 for Series Loss in 50 ohm systems; and Figure 12 for 75 ohm systems).

TECHNIQUE

Certain precautions are essential if reasonable accuracy is to be attained:

1. Accurate impedance measurement depends on accurate attenuation measurement. The comparison technique, diagrammed in Figure 3, is recommended. In this set-up the loss in one branch, containing the unknown, is compared with the loss in another, containing an accurate variable attenuator. This procedure is described in more detail in Tech. Newsletter #1. The use of carefully matched fixed attenuator pads with 5 or 10 db loss at the points indicated in Figure 3 will guard against impedance mismatch in the associated equipment and increase measurement accuracy. The use of such pads is only possible where the unknown has moderate loss, or where the measuring set-up includes a high-gain rf amplifier.

2. Any convenient physical arrangement may be used for a test jig provided that the system characteristic impedance is maintained. Two connectors mounted on a metal sheet can be used, as illustrated in Figure 4. The dimensions of the sheet should be several times the spacing of the connectors to minimize stray couplings from the circuit under test to the rest of the universe. Where the unknown circuit to be tested is itself coaxial (as where a short piece of cable is being tested) it is most convenient to use a "Tee" connector to attach the unknown to the junction.

3. Harmonics or other spurious components in the output of the sweep-frequency generator can cause errors, particularly when the unknown gives an attenuation maximum at resonance. Figure 5 illustrates some techniques that can be used to minimize the effects of spurious signals. When the measurements are being made within an octave band, a low pass filter cutting off at the upper end of the octave can be used to reduce harmonic errors (Figure 5A). Where there is a choice in regards to the circuit configuration, the use of those that give attenuation minima at resonance will help to reduce the effects of harmonics (Figure 5B). Finally, a tuned amplifier or receiver of moderate bandwidth (e.g. a television or radar receiver) can be used to minimize harmonics as well as to increase the gain of the system (Figure 5C).

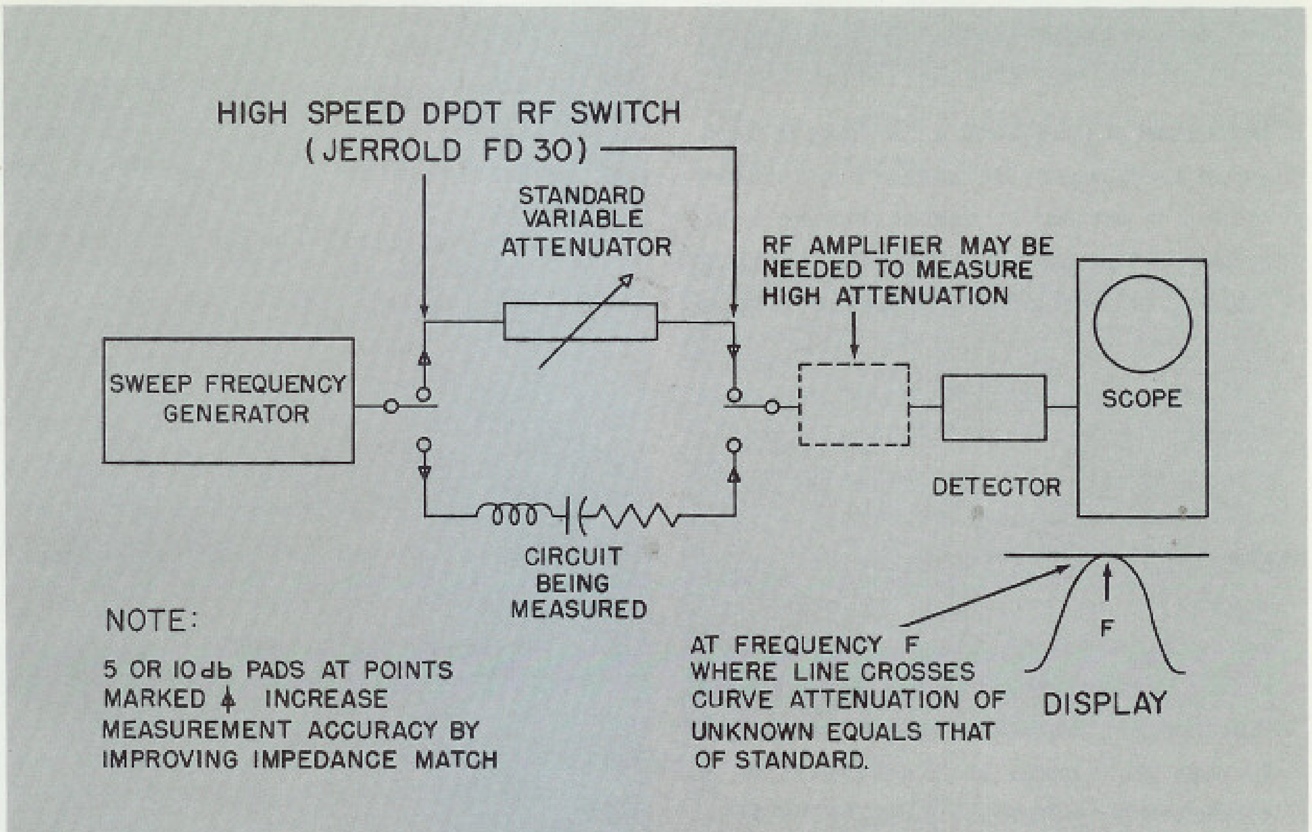


Figure 3—Technique for Accurate Attenuation Measurement

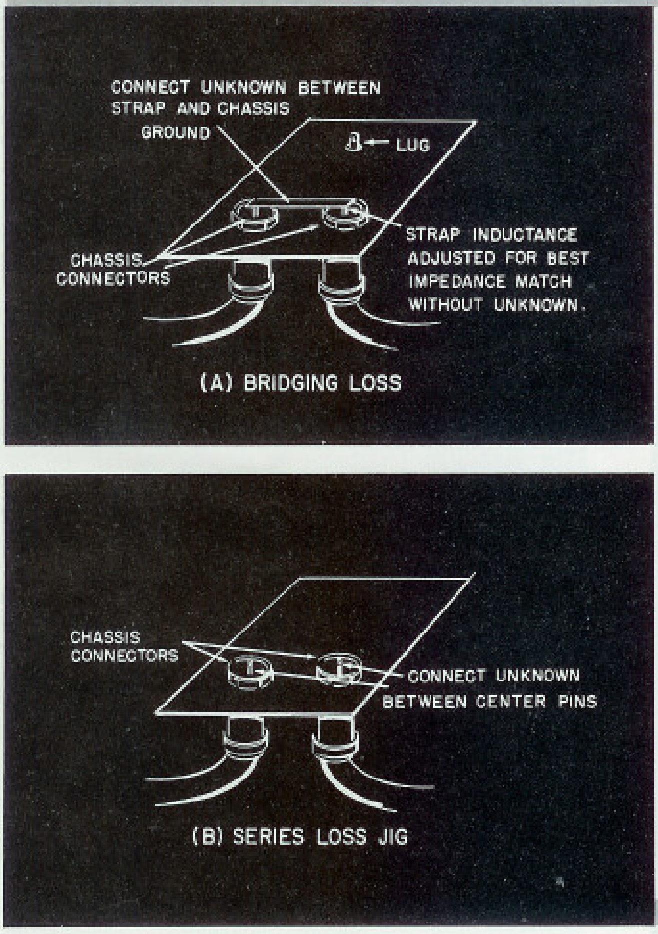


Figure 4—Test Jigs

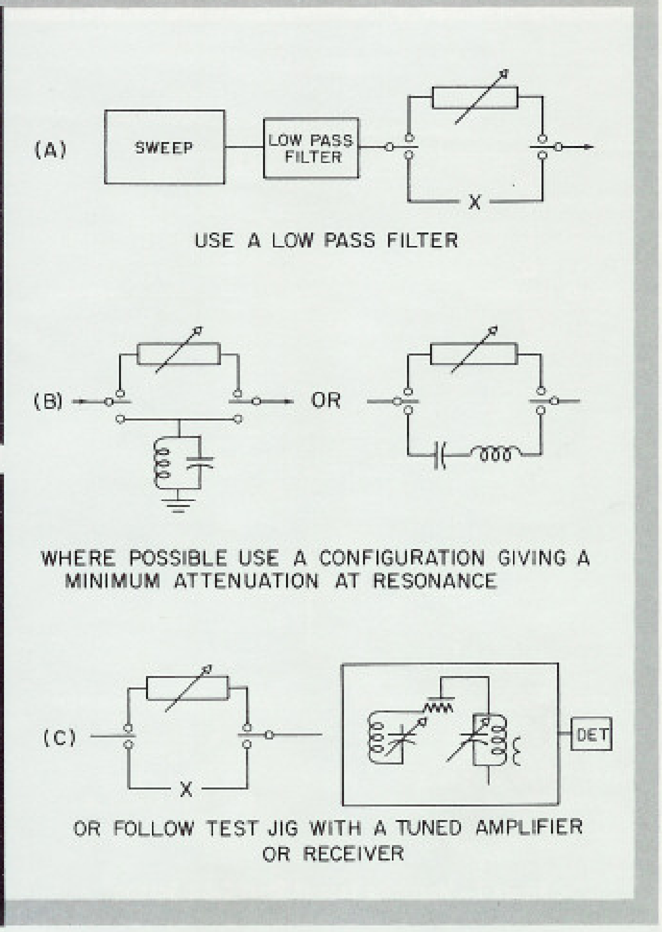


Figure 5—To Minimize Errors Due to Harmonics

APPLICATIONS AND EXAMPLES OF RESULTS

1. **Measurement of Tuned Circuit Q** (See Figure 6). A convenient way to measure the unloaded Q of a resonant circuit is to determine its resonant resistance by the bridging or series loss method. The Q is then found by comparing this resistance with the reactance of either reactor at resonance, calculated from a known L or C value. The choice as to the use of the bridging or series connection, and series or parallel resonance is determined by the impedance level and physical configuration of the circuit being tested. As an example, a coil space-wound with 15 turns of #16 copper wire was connected in series with a 15 mmfd TCZ ceramic capacitor. Its bridging loss as a series resonant circuit connected across a 75 ohm system was 31 db at 35 MC. From the chart (Figure 10) this indicates a series resistance of 1.1 ohms. The reactance of 15 mmfd at 35 MC is 303 ohms, so the circuit Q was $\frac{303}{1.1}$ or 275.

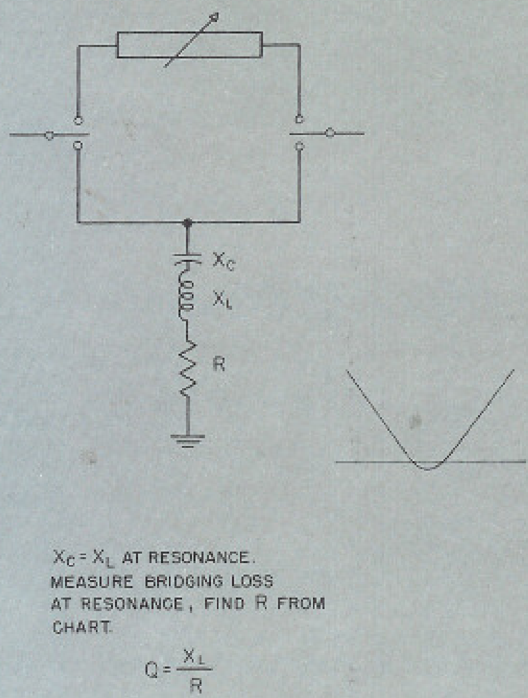


Figure 6—Measuring Q

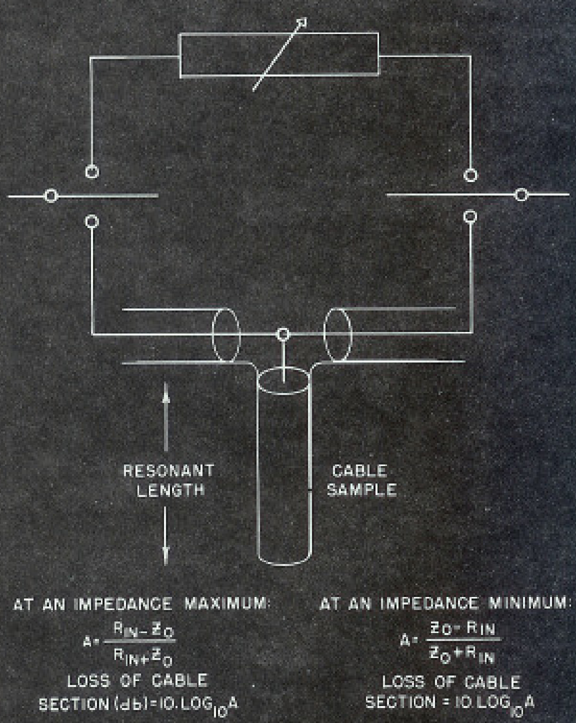


Figure 7—Measuring Loss of Short Cable Section

2. **Measurement of Attenuation of Short Cable Samples:**

The input resistance seen looking into a short section of transmission line which is open or shorted at the far end is a function of the loss of this line section. Using a coaxial measuring system, the input resistance of a resonant section of coaxial line can be measured most conveniently by the bridging method. (See Figure 7.) The measured resistance is directly related to the attenuation of that line section, and can be used to calculate it (see Appendix II, page 8).

As an example, the input resistance of a 3 foot section of RG-6/U open at the far end was measured by the bridging method and found to give a bridging loss of 31 db at its quarter-wave resonant frequency of 160 MC. From the chart (Figure 10, page 7) this is found to correspond to a resistance of 1.1 ohms. This gives an attenuation for this sample of 10 $\log_{10} \frac{75-1.1}{75+1.1}$ or 0.12 db; a loss of 4.0 db per 100 ft.

3. The Input Impedance of a Vacuum Tube:

Measuring the input impedance of a vacuum tube at high frequencies can be accomplished very simply by making this impedance part of a series resonant circuit by connecting an inductor in series with the grid terminal. Connecting the other end of the inductor as a bridging load across the measuring circuit allows rapid determination of the bridging loss, and thus of the total series resistance. (See Figure 8.) The measured resistance includes the losses of the inductor R_L as well as those of the tube Rin). R_L is small if a high Q inductor is chosen, but it can be measured quite accurately by replacing the tube with a high grade air trimmer set to give resonance at the same frequency, and measuring the bridging loss of the resulting circuit.

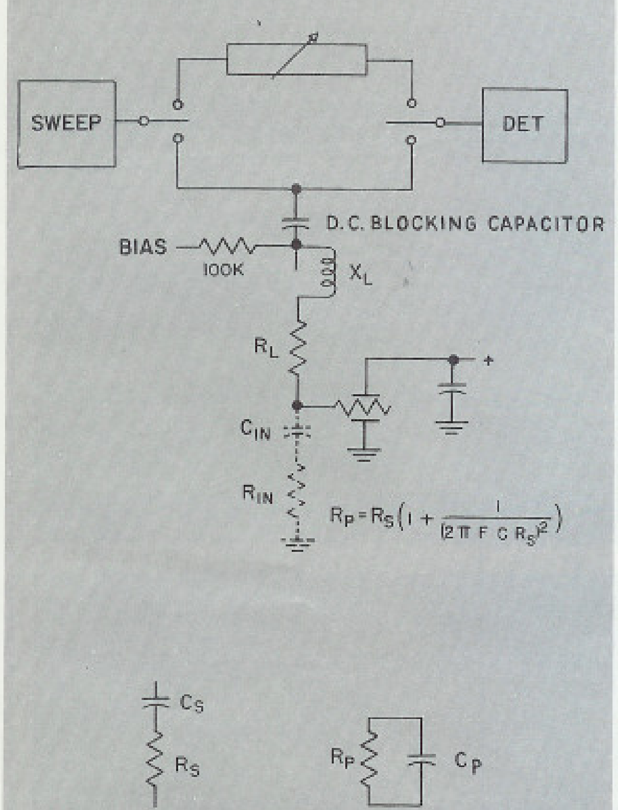


Figure 8—The Input Impedance of a Vacuum Tube

When measuring the input resistance of a tube by this method, it is notable that the measured resistance is only slightly affected by frequency. The results of measuring the input impedance of a 6CY5 tube with three different inductors are shown below.

Inductance	15.5 mh	1.08 mh	0.1 mh
Resonant Frequency	14.15 MC	54.3 MC	188 MC
Bridging Loss	12	14.3	13.3
Series R	12.5	9	10
Coil R	5.9	1.9	0.6
Tube R	6.6	7.1	9.4

It is commonly stated that the input resistance of a tube is approximately an inverse function of f^2. (See for example RCA Application Note AN-118 PP 8-9.) This statement is found to agree with the constant resistance described above when it is remembered that the R that is proportional to $\frac{1}{f^2}$ is the equivalent **parallel** resistance, while that which is approximately constant with frequency is the equivalent **series** resistance. Calling the former Rp and the latter Rs it can be easily shown that:

$$Rp = Rs\left(1 + \frac{1}{(2\pi fRsC)^2}\right)$$

If Rs is assumed constant, and Q is greater than 5 or so, this says that Rp is approximately equal to a constant divided by f^2.

Although both viewpoints are completely correct, it would seem more convenient to measure and think in terms of the relatively constant series resistive component, rather than the extremely variable parallel one.

Bridging Loss Nomographs

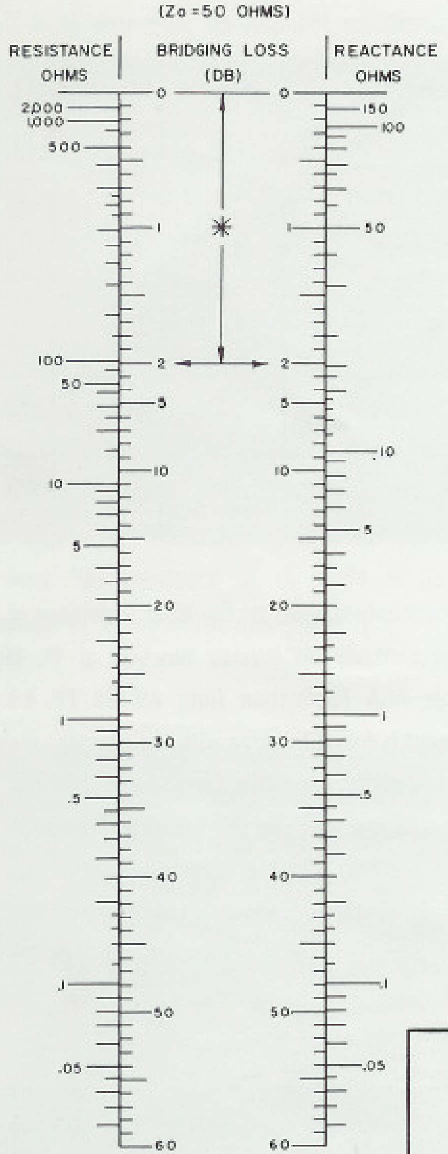


Figure 9

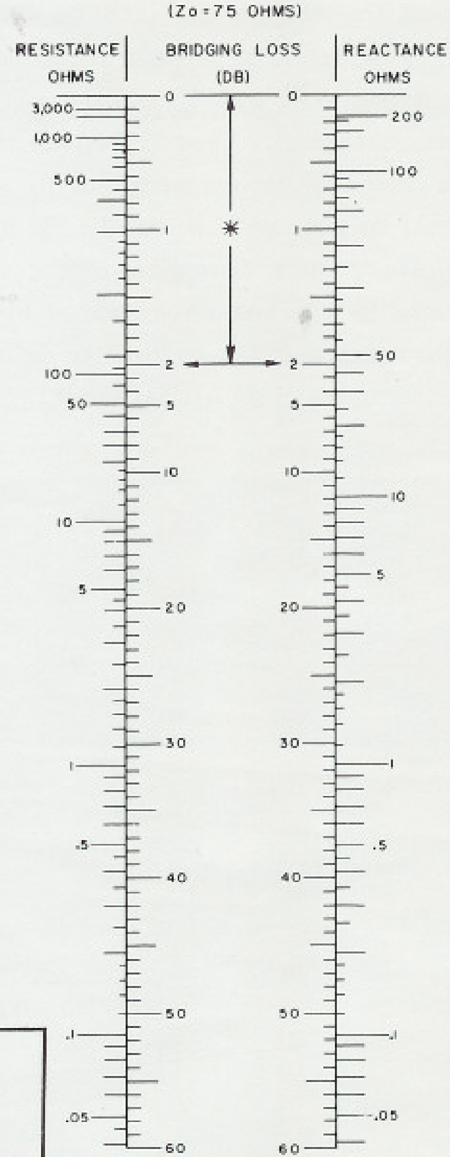
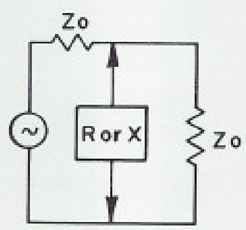


Figure 10

EQUIVALENT CIRCUIT



$$\text{DB LOSS} = 20 \log_{10}\left(1+\frac{Z_o}{2R}\right)$$

OR

$$\text{DB LOSS} = 10 \log_{10}\left(1+\frac{Z_o^2}{4X^2}\right)$$

NOTE:
✳ EXPANDED SCALE 0–2 DB

Series Loss Nomographs

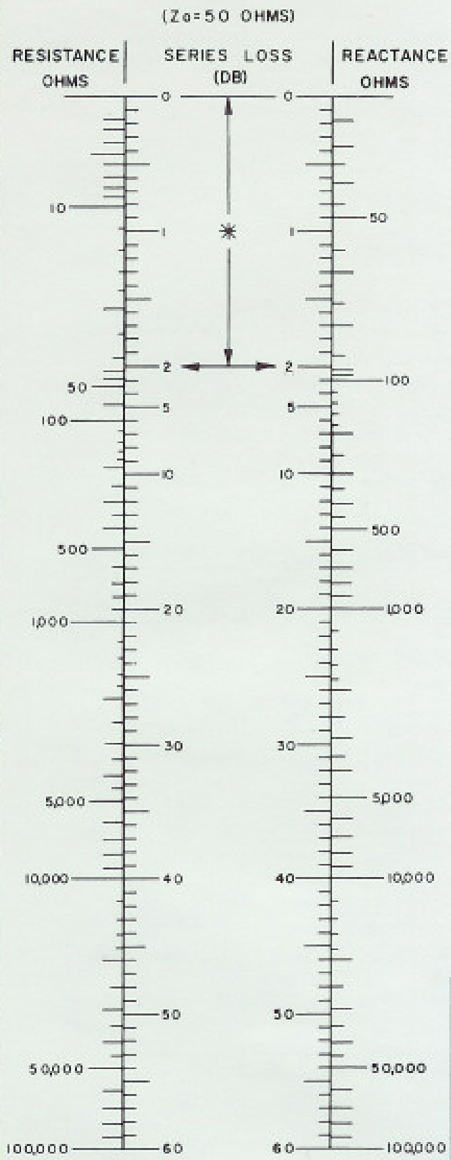


Figure 11

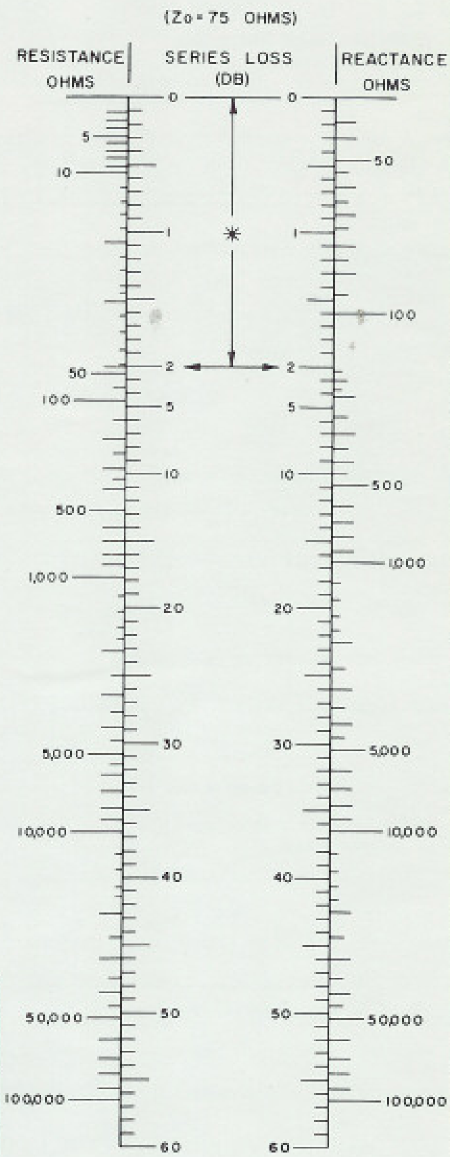
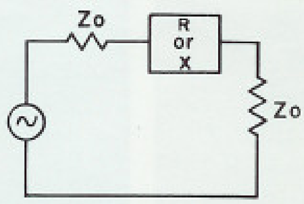


Figure 12

EQUIVALENT CIRCUIT



$$\text{DB LOSS} = 20 \log_{10}\left(1+\frac{R}{2Z_o}\right)$$

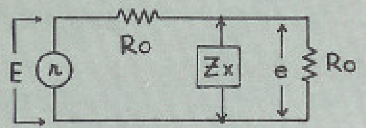
OR

$$\text{DB LOSS} = 10 \log_{10}\left(1+\frac{X^2}{4Z_o^2}\right)$$

NOTE:
* EXPANDED SCALE 0–2 DB

APPENDIX I

(a) The Calculation of Bridging Loss:
Equivalent circuit:



1. for Z_x = a pure resistance "Rx"

$$e = E\frac{\frac{R_o R_x}{R_o+R_x}}{R_o+\frac{R_o R_x}{R_o+R_x}} = E\frac{R_x}{R_o+2R_x}$$

$$e_{max} = \frac{E}{2} \qquad \frac{e_{max}}{e} = \frac{E/2}{\frac{E R_x}{R_o+2R_x}} = \frac{R_o}{2R_x}+1$$

insertion loss due to Rx =

$$20\log_{10}\left|\frac{e_{max}}{e}\right| = 20\log_{10}\left(\frac{R_o}{2R_x}+1\right)$$

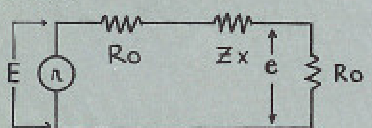
2. for Z_x = a pure reactance "jx"

$$e = E\frac{\frac{jR_o x}{R_o+jx}}{R_o+\frac{jR_o x}{R_o+jx}} = E\frac{jx}{R_o+2jx}$$

$$\frac{e_{max}}{e} = \frac{E/2}{\frac{Ejx}{R_o+2jx}} = \frac{R_o}{2jx}+1, \quad \left|\frac{e_{max}}{e}\right| = \sqrt{\left(\frac{R_o}{2x}\right)^2+1}$$

insertion loss due to x = $20\log_{10}\sqrt{\left(\frac{R_o}{2x}\right)^2+1}$

(b) The Calculation of Series Loss:
Equivalent circuit:



1. for Z_x = a pure resistance "Rx"

$$e = E\frac{R_o}{2R_o+R_x}$$

when Rx = o $e_{max} = E/2$, $\frac{e_{max}}{e} = \frac{E/2}{\frac{E R_o}{2R_o+R_x}} = \frac{R_x}{2R_o}+1$

insertion loss due to Rx = $20\log_{10}\left|\frac{e_{max}}{e}\right| = 20\log_{10}\left(\frac{R_x}{2R_o}+1\right)$

2. for Z_x = a pure reactance "jx"

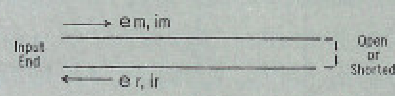
$$e = E\frac{R_o}{2R_o+jx} \qquad \frac{e_{max}}{e} = \frac{jx}{2R_o}+1$$

insertion loss due to x = $20\log_{10}\left|\frac{e_{max}}{e}\right| = 20\log_{10}\sqrt{\left(\frac{x}{2R_o}\right)^2+1}$

APPENDIX II

The Relation Between Input Resistance and Attenuation for a Resonant Length of Transmission Line

Characteristic impedance of section is Z_o



When the far end is open or short-circuited, the reflected voltage or current wave at the input terminals is reduced, compared with the main wave by twice the attenuation of the line expressed in db (since the reflected wave makes one trip down and one trip back to reach the input terminals). Calling the round trip attenuation "a" (as a current or voltage ratio:

$$e_r = a e_m, \; i_r = a i_m$$

(a) At a frequency where the input impedance is a maximum

$$R_{IN} = \frac{e_{max}}{i_{min}} = \frac{e_m+e_r}{i_m-i_r} = \frac{e_m}{i_m}\left(\frac{1+a}{1-a}\right) = Z_o\left(\frac{1+a}{1-a}\right)$$

$$Z_o + aZ_o = R_{IN} - aR_{IN}$$

$$\text{So} \quad a = \frac{R_{IN}-Z_o}{R_{IN}+Z_o}$$

$$\text{the one-way attenuation of this section (db)} = \frac{20\log_{10}a}{2}$$

$$= 10\log_{10}\frac{R_{IN}-Z_o}{R_{IN}+Z_o}$$

(b) At a frequency where the input impedance is a minimum

$$R_{IN} = \frac{e_{min}}{i_{max}} = \frac{e_m-e_r}{i_m+i_r} = \frac{e_m}{i_m} \times \frac{1-a}{1+a} = Z_o\frac{1-a}{1+a}$$

$$R_{IN} + aR_{IN} = Z_o - aZ_o, \text{ so } a = \frac{Z_o-R_{IN}}{Z_o+R_{IN}}$$

the one-way attenuation of this section (db) =

$$10\log_{10}\frac{Z_o-R_{IN}}{Z_o+R_{IN}}$$

NOTE — Additional copies of this paper and engineering assistance on the application of Measurements By Comparison may be obtained by contacting the Industrial Products Division, Jerrold Electronics Corporation, 15th and Lehigh, Philadelphia 32, Pa. Phone: BAldwin 6-3456 — Ext. 211.

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